Homework #3

1. Mark each statement True or False. Justify your answer.
(a) When an implication p $\Rightarrow$ q is used as a theorem, we refer to p as the antecedent.
(b) The contrapositive of p $\Rightarrow$ q is $\neg$ p $\Rightarrow$ $\neg$ q
(c) The inverse of p $\Rightarrow$ q is $\neg$ q $\Rightarrow$ $\neg$ p
(d) To prove "$\forall$ n, p(n)" is true, it takes only one example.
(e) To prove "$\exists$ n, p(n)" is true, it takes only one example.

2. Mark each statement True or False. Justify your answer.
(a) When an implication p $\Rightarrow$ q is used as a theorem, we refer to q as the conclusion.
(b) The converse of p $\Rightarrow$ q is q $\Rightarrow$ p
(c) To prove "$\forall$ n, p(n)" is false, it takes only one counterexample.
(d) To prove "$\exists$ n, p(n)" is false, it takes only one counterexample.

3. Write the contrapositive of each implication.
(a) If roses are red, then some violets are blue.
(b) A is not invertible, if there exists a nontrivial solution to Ax=0.
(c) If f is continuous and C is connected, then *f(C)* is connected.

4. Write the converse of each implication in Exercise 3.
(a) If roses are red, then some violets are blue.
(b) A is not invertible, if there exists a nontrivial solution to Ax=0
(c) If f is continuous and C is connected, then *f(C)* is connected.

5.Write the inverse of each implication in Exercise 3.
(a) If all roses are red, then some violets are blue.
(b) A is not invertible, if there exists a nontrivial solution to Ax=0
(c) If f is continuous and C is connected, then *f(C)* is connected.

6. Prove a counterexample for each statement.
(a) For every real number x, if $x^2 > 9$ then x > 3
(b) For every integer n, we have $n^3 \geq$ n.
(c) For all real numbers x $\geq$ 0, we have $x^2 \leq x^3$

(d) Every triangle is a right triangle.
(e) For every positive integer n, $n^2 + n + 41$ is prime.
(f) Every prime is an odd number.
(g) No integer greater than 100 is prime.
(h) 3^n+2 is prime for all positive integers n.
(i) For every integer n > 3, 3n is divisible by 6.
(j) If x and y are unequal positive integers and xy is perfect square, then x and y are perfect squares.
(k) For every real number x, there exists a real numbers such that xy=2
(l) The reciprocal of a real number x $\geq$1 is a real number y such that 0 < y < 1.
(m) No rational number satisfies the equation x^3+$(x-1)^2$= x^2+1
(n) No rational number satisfies the equation x^4+(1/x)-$\sqrt{x+1}$ =0

7. Prove the following:
(a) If p is odd and q is odd, then p+q is even.
(b) If p is odd and q is odd then pq is odd.
(c) If p is odd and q is odd, then p+3q is even.
(d) If p is odd and q is even, then p+q is odd.
(e) If p is even and q is even, then p+q is even.
(f) If p is even or q is even, then pq is even.
(g) If pq is odd, then p is odd and q is odd.
(h) If p^2 is even, then p is even.
(i) If p^2 is odd, then p is odd.